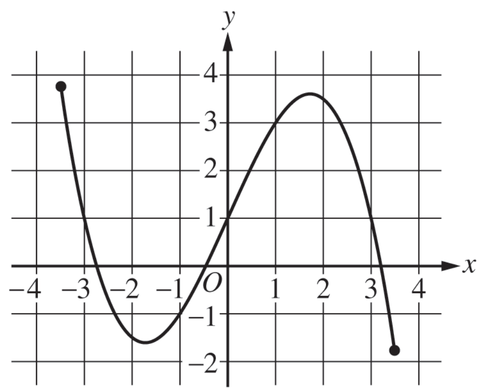
2024

AP

AP® Precalculus

Free-Response Questions



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

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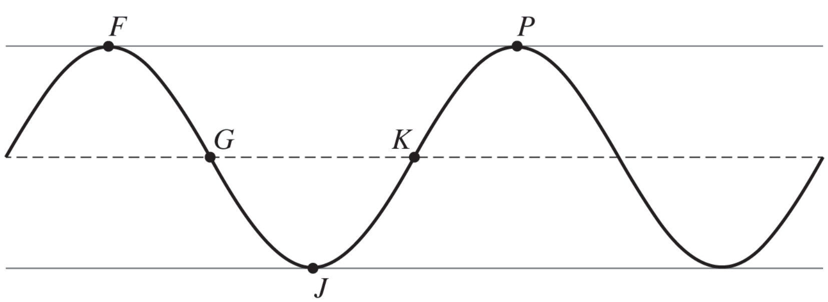
Note: Figure not drawn to scale.

3. The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

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2024 AP PreCalculus FRQ solutions - By Brenneman

① A i) $h(x) = g(f(x))$

$h(3) = g(f(3))$

$= g(1)$

$= 2.916 \cdot (0.7)^1$

$= 2.0412$

ii) $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$

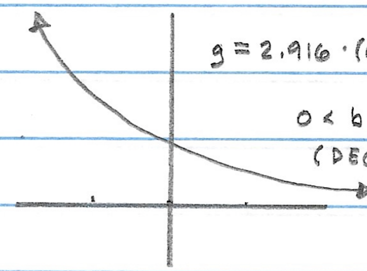
B i) $g(x) = 2$

$2.916 \cdot (0.7)^x = 2$

when $x = 1.05717$

(used F2 - Solve)

ii) $\lim_{x \to \infty} g(x) = 0$



$g = 2.916 \cdot (0.7)^x$

$0 < b < 1$

(DECAY)

C i) f does NOT have an inverse function

ii) Since f has output values that are mapped to more than one input value, f^{-1} is not a function.

($f(0) = 1$ & $f(3) = 1$ ⇒ $f^{-1}(1) = 0$ & $f^{-1}(1) = 3$)

② A i) $G(t) = a + b \cdot \ln(t+1)$

$G(0) = 40 = a + b \cdot \ln(0+1)$

$G(91) = 76 = a + b \cdot \ln(91+1)$

ii) $40 = a + b \cdot \ln(1)$; $76 = 40 + b \cdot \ln(92)$

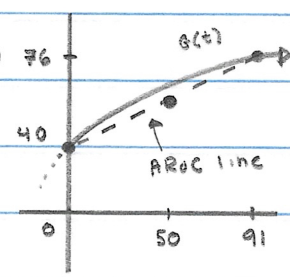
$40 = a + b(0)$ F2 → Solve

$a = 40$ $b = 7.96145$

B i) $\dfrac{G(91) - G(0)}{91 - 0} = \dfrac{76 - 40}{91}$

$= 0.395604 \dfrac{\text{thousands of games sold}}{1 \text{ day}}$

ii)



$\dfrac{G(50) - G(0)}{50 - 0} = 0.395604$

$G(50) = 50(0.395604) + 40$

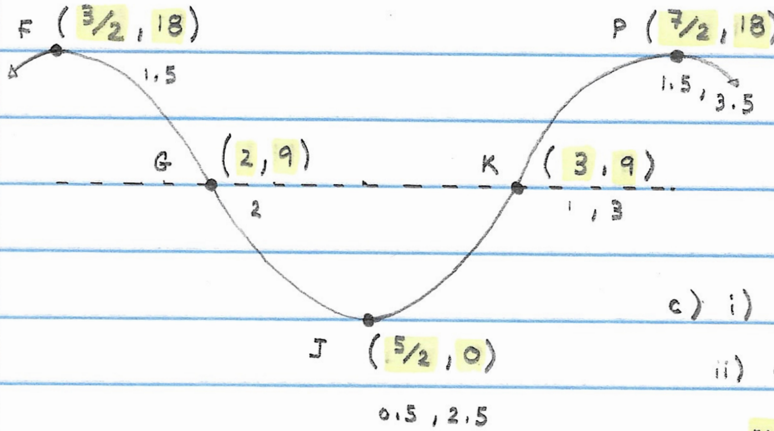
$= 59.7802$ (thousands)

c) i) The error in model G increases after $t = 91$ since the predicted value of $G(t)$ is increasing for $t > 91$, but the actual value of $G(t)$ is decreasing for $t > 91$.

(Error = Predicted − Actual)

iii) $A_t < G(t)$ for all t, $0 < t < 91$, since $G(t)$ is concave down on it's entire domain, & the secant line of A_t from $t = 0$ to $t = 91$ lies BENEATH $G(t)$ for all $t \in (0, 91)$.

③ A)



Period = $5/2 - 1/2 = 4/2 = 2$

Increments = $2/4 = 1/2 = 0.5$

c) i) h is pos & increasing (choice a)

ii) since h is concave down on (K, P), the rate of change of h is decreasing on (K, P).

B)

Amp = $(18-0)/2 = 9$	$a = 9$		$a = -9$
Mid = $(18+0)/2 = 9$	$b = \pi$	OR	$b = \pi$
Period = $2/1 = 2\pi/b \Rightarrow 2b = 2\pi \Rightarrow b = \pi$	$c = -3$		$c = -2$
Phase shift ⇒ since sine, choose midline	$d = 9$		$d = 9$

④ A i) $e^{x+3} = 10$

$\ln(e^{x+3}) = \ln(10)$

$(x+3)\cdot\ln(e) = \ln(10)$

$(x+3)(1) = \ln(10)$

$x = \ln(10) - 3$

ii) $\arcsin(x/2) = \pi/4$

$\sin(\arcsin(x/2)) = \sin(\pi/4)$

$x/2 = \sqrt{2}/2$

$2(x/2) = 2(\sqrt{2}/2)$

$x = \sqrt{2}$

$\sin^2 + \cos^2 = 1$

$\cos^2 = 1 - \sin^2$

B i) $j(x) = \log(8x^5) + \log(2x^2) - 9\log(x)$

$= \log(8x^5 \cdot 2x^2) - \log(x)^9$

$= \log(16x^7) - \log(x^9)$

$= \log(16x^7/x^9)$

$= \log_{10}(16/x^2)$, $x > 0$

ii) $K(x) = \left(\dfrac{1-\sin^2 x}{\sin x}\right) \cdot \sec x$

$= \left(\dfrac{\cos^2 x}{\sin x}\right) \cdot \dfrac{1}{\cos x} = \dfrac{\cos x}{\sin x} = \cot x = 1/\tan x$

c) $\cos^{-1}(\tan(2x)) = 0$

$\cos(\cos^{-1}(\tan(2x))) = \cos(0)$

$\tan(2x) = 1$

$\tan^{-1}(\tan(2x)) = \tan^{-1}(1)$

$\rightarrow 2x = \pi/4 + \pi n, n \in \mathbb{Z}$

$1/2\,(2x) = 1/2\,(\pi/4 + \pi n, n \in \mathbb{Z})$

$x = \pi/8 + \pi n/2, n \in \mathbb{Z}$

OR

$x = \pi/8 + \pi n, n \in \mathbb{Z}$ and $x = 5\pi/8 + \pi n, n \in \mathbb{Z}$